

## HYPOTHESIS TESTS FOR PROPORTIONS ON THE COMPUTER

Hypothesis tests for proportions are so easy and natural that many statistics packages don't offer special commands for them. Most statistics programs want to know the "success" and "failure" status for each case. Usually these are given as 1 or 0, but they might be category names like "yes" and "no." Often we just know the proportion of successes,  $\hat{p}$ , and the total count,  $n$ . Computer packages don't usually deal naturally with summary data like this, but the statistics routines found on many graphing calculators do. These calculators allow you to test hypotheses from summaries of the data—usually, all you need to enter are the number of successes and the sample size.

## EXERCISES

- Hypotheses.** Write the null and alternative hypotheses you would use to test each of the following situations:

  - A governor is concerned about his "negatives"—the percentage of state residents who express disapproval of his job performance. His political committee pays for a series of TV ads, hoping that they can keep the negatives below 30%. They will use follow-up polling to assess the ads' effectiveness.
  - Is a coin fair?
  - Only about 20% of people who try to quit smoking succeed. Sellers of a motivational tape claim that listening to the recorded messages can help people quit.
- More hypotheses.** Write the null and alternative hypotheses you would use to test each situation.

  - In the 1950s only about 40% of high school graduates went on to college. Has the percentage changed?
  - 20% of cars of a certain model have needed costly transmission work after being driven between 50,000 and 100,000 miles. The manufacturer hopes that a re-design of a transmission component has solved this problem.
  - We field-test a new-flavor soft drink, planning to market it only if we are sure that over 60% of the people like the flavor.
- Negatives.** After the political ad campaign described in Exercise 1a, pollsters check the governor's negatives. They test the hypothesis that the ads produced no change against the alternative that the negatives are now below 30% and find a P-value of 0.22. Which conclusion is appropriate? Explain.

  - There's a 22% chance that the ads worked.
  - There's a 78% chance that the ads worked.
  - There's a 22% chance that their poll is correct.
  - There's a 22% chance that natural sampling variation could produce poll results like these if there's really no change in public opinion.
- Dice.** The seller of a loaded die claims that it will favor the outcome 6. We don't believe that claim, and roll the die 200 times to test an appropriate hypothesis. Our P-value turns out to be 0.03. Which conclusion is appropriate? Explain.

  - There's a 3% chance that the die is fair.
  - There's a 97% chance that the die is fair.
  - There's a 3% chance that a loaded die could randomly produce the results we observed, so it's reasonable to conclude that the die is fair.
  - There's a 3% chance that a fair die could randomly produce the results we observed, so it's reasonable to conclude that the die is loaded.
- Relief.** A company's old antacid formula provided relief for 70% of the people who used it. The company tests a new formula to see if it is better and gets a P-value of 0.27. Is it reasonable to conclude that the new formula and the old one are equally effective? Explain.
- Cars.** A survey investigating whether the proportion of today's high school seniors who own their own cars is higher than it was a decade ago finds a P-value of 0.017. Is it reasonable to conclude that more high-schoolers have cars? Explain.
- He cheats!** A friend of yours claims that when he tosses a coin he can control the outcome. You are skeptical and want him to prove it. He tosses the coin, and you call heads; it's tails. You try again and lose again.

  - Do two losses in a row convince you that he really can control the toss? Explain.
  - You try a third time, and again you lose. What's the probability of losing three tosses in a row if the process is fair?
  - Would three losses in a row convince you that your friend cheats? Explain.
  - How many times in a row would you have to lose in order to be pretty sure that this friend really can control the toss? Justify your answer by calculating a probability and explaining what it means.
- Candy.** Someone hands you a box of a dozen chocolate-covered candies, telling you that half are vanilla creams and the other half peanut butter. You pick candies at random and discover the first three you eat are all vanilla.

- a) If there really were 6 vanilla and 6 peanut butter candies in the box, what is the probability that you would have picked three vanillas in a row?
- b) Do you think there really might have been 6 of each? Explain.
- c) Would you continue to believe that half are vanilla if the fourth one you try is also vanilla? Explain.

9. **Cell phones.** Many people have trouble setting up all the features of their cell phones, so a company has developed what it hopes will be easier instructions. The goal is to have at least 96% of customers succeed. The company tests the new system on 200 people, of whom 188 were successful. Is this strong evidence that the new system fails to meet the company's goal? A student's test of this hypothesis is shown. How many mistakes can you find?

$$H_0: \hat{p} = 0.96$$

$$H_A: \hat{p} \neq 0.96$$

$$\text{SRS}, 0.96(200) > 10$$

$$\frac{188}{200} = 0.94; \quad SD(\hat{p}) = \sqrt{\frac{(0.94)(0.06)}{200}} = 0.017$$

$$z = \frac{0.96 - 0.94}{0.017} = 1.18$$

$$P = P(z > 1.18) = 0.12$$

There is strong evidence the new instructions don't work.

10. **Got milk?** In November 2001, the *Ag Globe Trotter* newsletter reported that 90% of adults drink milk. A regional farmers' organization planning a new marketing campaign across its multicounty area polls a random sample of 750 adults living there. In this sample, 657 people said that they drink milk. Do these responses provide strong evidence that the 90% figure is not accurate for this region? Correct the mistakes you find in a student's attempt to test an appropriate hypothesis.

$$H_0: \hat{p} = 0.9$$

$$H_A: \hat{p} < 0.9$$

$$\text{SRS}, 750 > 10$$

$$\frac{657}{750} = 0.876; \quad SD(\hat{p}) = \sqrt{\frac{(0.88)(0.12)}{750}} = 0.012$$

$$z = \frac{0.876 - 0.90}{0.012} = -2$$

$$P = P(z > -2) = 0.977$$

There is more than a 97% chance that the stated percentage is correct for this region.

11. **Dowsing.** In a rural area, only about 30% of the wells that are drilled find adequate water at a depth of 100 feet or less. A local man claims to be able to find water by "dowsing"—using a forked stick to indicate where the well should be drilled. You check with 80 of his customers and find that 27 have wells less than 100 feet deep. What do you conclude about his claim?
- Write appropriate hypotheses.
  - Check the necessary assumptions.
  - Perform the mechanics of the test. What is the P-value?
  - Explain carefully what the P-value means in context.
  - What's your conclusion?

12. **Abnormalities.** In the 1980s it was generally believed that congenital abnormalities affected about 5% of the nation's children. Some people believe that the increase in the number of chemicals in the environment has led to an increase in the incidence of abnormalities. A recent study examined 384 children and found that 46 of them showed signs of an abnormality. Is this strong evidence that the risk has increased?

- Write appropriate hypotheses.
- Check the necessary assumptions.
- Perform the mechanics of the test. What is the P-value?
- Explain carefully what the P-value means in context.
- What's your conclusion?
- Do environmental chemicals cause congenital abnormalities?

13. **Absentees.** The National Center for Education Statistics monitors many aspects of elementary and secondary education nationwide. Their 1996 numbers are often used as a baseline to assess changes. In 1996 34% of students had not been absent from school even once during the previous month. In the 2000 survey, responses from 8302 students showed that this figure had slipped to 33%. Officials would, of course, be concerned if student attendance were declining. Do these figures give evidence of a change in student attendance?

- Write appropriate hypotheses.
- Check the assumptions and conditions.
- Perform the test and find the P-value.
- State your conclusion.
- Do you think this difference is meaningful? Explain.

14. **Educated mothers.** The National Center for Education Statistics monitors many aspects of elementary and secondary education nationwide. Their 1996 numbers are often used as a baseline to assess changes. In 1996, 31% of students reported that their mothers had graduated from college. In 2000, responses from 8368 students found that this figure had grown to 32%. Is this evidence of a change in education level among mothers?

- Write appropriate hypotheses.
- Check the assumptions and conditions.
- Perform the test and find the P-value.
- State your conclusion.
- Do you think this difference is meaningful? Explain.

15. **Contributions, please, part II.** In Exercise 19.15 you learned that the Paralyzed Veterans of America is a philanthropic organization that relies on contributions. They send free mailing labels and greeting cards to potential donors on their list and ask for a voluntary contribution. To test a new campaign, the organization recently sent letters to a random sample of 100,000 potential donors and received 4781 donations. They've had a contribution rate of 5% in past campaigns, but a staff member worries that the rate will be lower if they run this campaign as currently designed.

- What are the hypotheses?
- Are the assumptions and conditions for inference met?
- Do you think the rate would drop? Explain.

16. **Take the offer, part II.** In Exercise 19.16 you learned that First USA, a major credit card company, is planning a new offer for their current cardholders. First USA will give double airline miles on purchases for the next 6 months if the cardholder goes online and registers for this offer. To test the effectiveness of this campaign, the company recently sent out offers to a random sample of 50,000 cardholders. Of those, 1184 registered. A staff member suspects that the success rate for the full campaign will be comparable to the standard 2% rate that they are used to seeing in similar campaigns. What do you predict?
- What are the hypotheses?
  - Are the assumptions and conditions for inference met?
  - Do you think the rate would change if they use this fundraising campaign? Explain.
17. **Law School.** According to the Law School Admission Council, in the fall of 2006, 63% of law school applicants were accepted to some law school.<sup>4</sup> The training program *LSATisfaction* claims that 163 of the 240 students trained in 2006 were admitted to law school. You can safely consider these trainees to be representative of the population of law school applicants. Has *LSATisfaction* demonstrated a real improvement over the national average?
- What are the hypotheses?
  - Check the conditions and find the P-value.
  - Would you recommend this program based on what you see here? Explain.
18. **Med School.** According to the Association of American Medical Colleges, only 46% of medical school applicants were admitted to a medical school in the fall of 2006.<sup>5</sup> Upon hearing this, the trustees of Striving College expressed concern that only 77 of the 180 students in their class of 2006 who applied to medical school were admitted. The college president assured the trustees that this was just the kind of year-to-year fluctuation in fortunes that is to be expected and that, in fact, the school's success rate was consistent with the national average. Who is right?
- What are the hypotheses?
  - Check the conditions and find the P-value.
  - Are the trustees right to be concerned, or is the president correct? Explain.
19. **Pollution.** A company with a fleet of 150 cars found that the emissions systems of 7 out of the 22 they tested failed to meet pollution control guidelines. Is this strong evidence that more than 20% of the fleet might be out of compliance? Test an appropriate hypothesis and state your conclusion. Be sure the appropriate assumptions and conditions are satisfied before you proceed.
20. **Scratch and dent.** An appliance manufacturer stockpiles washers and dryers in a large warehouse for shipment to retail stores. Sometimes in handling them the appliances get damaged. Even though the damage may be minor, the company must sell those machines at drastically reduced prices. The company goal is to keep the level of damaged machines below 2%. One day an inspector randomly checks 60 washers and finds that 5 of them have scratches or dents. Is this strong evidence that the warehouse is failing to meet the company goal? Test an appropriate hypothesis and state your conclusion. Be sure the appropriate assumptions and conditions are satisfied before you proceed.
21. **Twins.** In 2001 a national vital statistics report indicated that about 3% of all births produced twins. Is the rate of twin births the same among very young mothers? Data from a large city hospital found that only 7 sets of twins were born to 469 teenage girls. Test an appropriate hypothesis and state your conclusion. Be sure the appropriate assumptions and conditions are satisfied before you proceed.
22. **Football 2006.** During the 2006 season, the home team won 136 of the 240 regular-season National Football League games. Is this strong evidence of a home field advantage in professional football? Test an appropriate hypothesis and state your conclusion. Be sure the appropriate assumptions and conditions are satisfied before you proceed.
23. **WebZine.** A magazine is considering the launch of an online edition. The magazine plans to go ahead only if it's convinced that more than 25% of current readers would subscribe. The magazine contacted a simple random sample of 500 current subscribers, and 137 of those surveyed expressed interest. What should the company do? Test an appropriate hypothesis and state your conclusion. Be sure the appropriate assumptions and conditions are satisfied before you proceed.
24. **Seeds.** A garden center wants to store leftover packets of vegetable seeds for sale the following spring, but the center is concerned that the seeds may not germinate at the same rate a year later. The manager finds a packet of last year's green bean seeds and plants them as a test. Although the packet claims a germination rate of 92%, only 171 of 200 test seeds sprout. Is this evidence that the seeds have lost viability during a year in storage? Test an appropriate hypothesis and state your conclusion. Be sure the appropriate assumptions and conditions are satisfied before you proceed.
25. **Women executives.** A company is criticized because only 13 of 43 people in executive-level positions are women. The company explains that although this proportion is lower than it might wish, it's not surprising given that only 40% of all its employees are women. What do you think? Test an appropriate hypothesis and state your conclusion. Be sure the appropriate assumptions and conditions are satisfied before you proceed.
26. **Jury.** Census data for a certain county show that 19% of the adult residents are Hispanic. Suppose 72 people are called for jury duty and only 9 of them are Hispanic. Does this apparent underrepresentation of Hispanics call into question the fairness of the jury selection system? Explain.

<sup>4</sup> As reported by the Cornell office of career services in their *Class of 2006 Postgraduate Report*.

<sup>5</sup> *Ibid.*

27. **Dropouts.** Some people are concerned that new tougher standards and high-stakes tests adopted in many states have driven up the high school dropout rate. The National Center for Education Statistics reported that the high school dropout rate for the year 2004 was 10.3%. One school district whose dropout rate has always been very close to the national average reports that 210 of their 1782 high school students dropped out last year. Is this evidence that their dropout rate may be increasing? Explain.
28. **Acid rain.** A study of the effects of acid rain on trees in the Hopkins Forest shows that 25 of 100 trees sampled exhibited some sort of damage from acid rain. This rate seemed to be higher than the 15% quoted in a recent *Environmetrics* article on the average proportion of damaged trees in the Northeast. Does the sample suggest that trees in the Hopkins Forest are more susceptible than trees from the rest of the region? Comment, and write up your own conclusions based on an appropriate confidence interval as well as a hypothesis test. Include any assumptions you made about the data.
29. **Lost luggage.** An airline's public relations department says that the airline rarely loses passengers' luggage. It further claims that on those occasions when luggage is lost, 90% is recovered and delivered to its owner within 24 hours. A consumer group that surveyed a large number of air travelers found that only 103 of 122 people who lost luggage on that airline were reunited with the missing items by the next day. Does this cast doubt on the airline's claim? Explain.
30. **TV ads.** A start-up company is about to market a new computer printer. It decides to gamble by running commercials during the Super Bowl. The company hopes that name recognition will be worth the high cost of the ads. The goal of the company is that over 40% of the public recognize its brand name and associate it with computer equipment. The day after the game, a pollster contacts 420 randomly chosen adults and finds that 181 of them know that this company manufactures printers. Would you recommend that the company continue to advertise during Super Bowls? Explain.
31. **John Wayne.** Like a lot of other Americans, John Wayne died of cancer. But is there more to this story? In 1955 Wayne was in Utah shooting the film *The Conqueror*. Across the state line, in Nevada, the United States military was testing atomic bombs. Radioactive fallout from those tests drifted across the filming location. A total of 46 of the 220 people working on the film eventually died of cancer. Cancer experts estimate that one would expect only about 30 cancer deaths in a group this size.
- a) Is the death rate among the movie crew unusually high?
- b) Does this prove that exposure to radiation increases the risk of cancer?
32. **AP Stats.** The College Board reported that 60% of all students who took the 2006 AP Statistics exam earned scores of 3 or higher. One teacher wondered if the performance of her school was different. She believed that year's students to be typical of those who will take AP Stats at that school and was pleased when 65% of her 54 students achieved scores of 3 or better. Can she claim that her school is different? Explain.



### JUST CHECKING Answers

1. You can't conclude that the null hypothesis is true. You can conclude only that the experiment was unable to reject the null hypothesis. They were unable, on the basis of 12 patients, to show that aspirin was effective.
2. The null hypothesis is  $H_0: p = 0.75$ .
3. With a P-value of 0.0001, this is very strong evidence against the null hypothesis. We can reject  $H_0$  and conclude that the improved version of the drug gives relief to a higher proportion of patients.
4. The parameter of interest is the proportion,  $p$ , of all delinquent customers who will pay their bills.  $H_0: p = 0.30$  and  $H_A: p > 0.30$ .
5. The very low P-value leads us to reject the null hypothesis. There is strong evidence that the DVD is more effective in getting people to start paying their debts than just sending a letter had been.
6. All we know is that there is strong evidence to suggest that  $p > 0.30$ . We don't know how much higher than 30% the new proportion is. We'd like to see a confidence interval to see if the new method is worth the cost.